Government of Karnataka

Forest Department

Recruitment Examination (Main) for Range Forest Officers Training Course-2007

07 November-2007 (10.30am-1.30pm)

Option Paper : MATHEMATICS

Max, Marks: 100 Time : 3 Hours

All Section are compulsory

PART-A (1 Mark each)

1) Answer all questions

a) Let V be the vector space of all functions from R into R. Show that W={f: f(-x)=-f(x)} is a subspace of V.

b) Find the value of $\lim_{x\to\infty}\left(\frac{x-1}{x-1}\right)^x$

c) Determine the value of f(0) so that the function

$$f(x) = \frac{(a^2-ax+x^2)^{1/2} - (a^2+ax+x^2)^{1/2}}{(a+x)^{1/2} - (a-x)^{1/2}}$$

is continuous for all x.

d) If a and b are elements of a group G and if

$$a^2 = 1, a^{-1}b^2a = ba \quad \text{then prove that} \quad b = a$$

e) Show that σ = ▯▯ $\begin{pmatrix}1 & 2 & 3\\ 3 & 1 & 2\end{pmatrix}$ *is even.*

f) Prove that $L\,(sinat) = \frac{a}{s^2+a^2}\ for\ s > 0.$

g) Find the locus of the points Z satisfying arg $\left(\frac{z-1}{z+1}\right) = \frac{\pi}{3}$

h) Find the equation of the straight line joining the points (-2,1,3) and (3,1,-2).

i) If $\begin{vmatrix} x & x^2 & 1+x^2\\ y & y^2 & 1+y^2\\ z & z^2 & 1+z^2\end{vmatrix} = 0\ then\ prove\ that$

The value of xyz is =1.

j) Show that the rank of the matrix $\begin{pmatrix}3 & 2\\ 6 & 4\end{pmatrix}$ $is\ 1.$

k) Find the distance between the two planes 2x-2y+z+3=0 and 4x-4y+2z+5=0.

l) Show that the series 2-2+2-2+ ……… oscillates.

m) Find an integrating factor of $x\,(1-x^2)\mathrm{dy}+(2x^2\mathrm{y}-\mathrm{y}-\mathrm{a}x^3)\mathrm{dx}=0$

n) If $f=\mathrm{r}\cos\theta+\tan\theta\ find\ grad f\ in\ polar\ co-ordinates$

o) The length of a simple pendulum in increased by 44%. Show that the percentage increase in its time period is 20 % .

p) Prove that $y=cx^{-1}+\mathrm{d}$ is a solution of $\frac{\mathrm{d^2y}}{\mathrm{dx^2}}+\frac{2}{\mathrm{x}}\frac{\mathrm{dy}}{\mathrm{dx}}=0$

q) Find f_x and f_y when f (x,y) = x^2+y^2

r) Find the n^{th} derivative of $(ax+b)^m$

s) For any integer a show that $a^3=0{,}1\ or\ 6\ (mod 7)$

t) If f(t) = $(t-t^2)$ i + t^3j-3k then find $\int f(t)dt$.

PART-B (3 Mark each)

2) Solve the linear congruence 9x ≡21 (mod 30)

OR

Solve x^2 (y+1)dx + Y^2(x-1) dy = 0.

3) Let G and H be two groups and let η: G→H be a homomorphism where N=Ker η. Show that G — $\frac{G}{N}$.

OR

Show that three concurrent lines with direction cosines l_1, m_1, n_1, l_2 m_2 n_2 and l_3, m_3, n_3 are coplanar if $\begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix}=0$

4) Find the differential equation of the family of curves

$$xy=Ae^x+\ Be^{-x}+x^2 \text{ for different values of A and B.}$$

OR

Solve $y^{111}\ -3y^{11}\ +3y^1 - y=\ t^2\,e^t\ where\ y\ (0)=1, y^1\,(0)=0, y^{11}\,(0)=-2.$

5) Show that any two bases of a finite dimensional vector space have the same number of vectors.

OR

Prove that a finite integral domain is a field.

6) Let A= $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 6 & -11 & 6 \end{pmatrix}$ be in F_3 where the characteristic of F is not 2. Show that

$$A^3 - 6A^2 + 11A - 6 = 0$$

OR

If $\Delta x = 0.005$, $\Delta y = 0.001$ be the absolute errors in x = 2.11 and y = 4.15, find the relative error in the computation of x + y.

PART-C (12 Mark each)

7) Use Fermat's theorem to verify that 17 divides $11^{104} + 1$.

Test for convergence or divergence of the series

$$\sum_{n=1}^{\infty} \left[\sqrt{n^4+1} - \sqrt{n^4-1}\right]$$

OR

Find the maximum and minimum values of sinx + cos 2x

Solve $x^3-6x-9=0$ using cordon's method.

8) Prove that the lines $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$ *and* $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z-10}{8}$ are coplanar. Find also the point of intersection, find $L^{-1}\left[\frac{1}{s^2+1)^2}\right]$

OR

Show that the function e^x (cosy + I siny) is holomorphic and find its derivative.

Let R be a commutative ring with unity and let M be an ideal of R. Prove that M is a maximal ideal of R if and only if $\frac{R}{M}$ is a field

9) If a function f(z) is analytic for all values of Z and is bonded then show that f is a constant. Prove that grad (uv) = u grad v + v grad u.

OR

State and prove green's theorem in the plane. Using this evaluate

$$\oint_c (x^2 - \cosh y)\, dx + (y + \sin x) dy$$

Where c is the rectangle with vertices (0,0), $(\pi.0)$ $(\pi,1) and$ $(0,1)$

10) Let V be a vector space over a field F, if V_1 ……….V_n is a basis of V over F and if W_1 …………… W_m in V are linearly independent over F then show that m$\leq n$.

Find the bilinear transformation which transforms z=2,1,0 into w=1,0,i.

OR

Solve by the method of variation of parameters $\frac{d^2y}{dx^2} + y = cosec\ x$.

Find the rank of matrix $\begin{pmatrix} 1 & 1 & 2 & 3 \\ 1 & 2 & 3 & 4 \\ 1 & -3 & -2 & -1 \end{pmatrix}$

11) Solve x + y + z = 1

2x + 3y + 4z = 1

x – y – z =0

Using cramer's rule.

Starting from gauss interpolation formula derive Bessel's central difference formula.

OR

Describe how to use trapezoidal and simpson's $^1/_3$ rule to evaluate $\pi = \int_0^1 \frac{4dx}{(1+x^2)}$

Find the inverse of the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

Using cayley – Hamilton theorem.